Adaptive Extremum Seeking Control of Continuous Stirred-Tank Bioreactors

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An adaptive extremum-seeking control scheme for continuous stirred-tank bioreactors is proposed which utilizes the structure information of the kinetics of bioreactors to construct a seeking algorithm that drives the system states to the desired setpoints that optimize the value of an objective function. Lyapunov's stability theorem is used in the design of the extremum-seeking controller stucture and the development of the parameter learning laws. Numerical simulations illustrate the effectiveness of this approach.

Introduction

Most adaptive control schemes documented in the literature (Landau, 1979; Goodwin and Sin, 1984; Astrom and Wittenmark, 1995; Narendra and Annaswamy, 1989; Ioannou and Sun, 1996; Krstic et al., 1995) are developed for regulation to known set points or the tracking of known reference trajectories. In some applications, however, the control objective could be to optimize an objective function that can be a function of unknown parameters, or to select the desired states to keep a performance function at its extremum value. Self-optimizing control and extremum-seeking control are two methods to handle these kinds of optimization problems. The goal of self-optimizing control is to find a set of controller variables which, when kept at constant setpoints, indirectly lead to near-optimal operation with acceptable loss (Findeisen et al., 1980; Morari et al., 1980; Skogestad, 2000). The task of extremum seeking is to find the operating setpoints that maximize or minimize an objective function. Since the early research work on extremum control in the 1920s (Leblanc, 1922), many successful applications of extremum-control approaches have been reported, for example, fuel-flow control to achieve maximum pressure (Vasu, 1957), combustion process control for IC engines and gas furnaces (Astrom and Wittenmark, 1995; Sternby, 1980), and antilock braking system control (Drkunov et al., 1995). Although a large amount of research efforts has been done by Morosanov (1957), Ostrovskii (1957), Blackman (1962), Frey et al. (1966), Jacobs and Shering (1968), Pevozvanski (1960), a solid theoretical foundation has not yet been established for the stability and performance of extremum seeking control.

Recently, Krstic et al. (2000) and Krstic and Wang (2000) presented several extremum control schemes and a stability analysis for extremum seeking of linear unknown systems and a class of general nonlinear systems (Krstic, 2000; Krstic and Wang, 2000; Krstic and Deng, 1998). Applications of these approaches have been reported for the maximization of pressure rise in an axial-flow compressor (Wang et al., 1998). In Speyer et al. (2000), a peak-seeking controller is designed to drive a linear system operating an unknown setpoint that maximizes the specified performance function. Using a modified Kalman filter to estimate the derivatives of the performance function, the peaking-seeking scheme proposed in Speyer et al. (2000) has been applied to formation flight to find the "best flight position" where drag is minimized (Chichka et al., 1999; Banavar et al., 2000). It is also worth noting that extremum control with Monod kinetics has also been considered and studied in (Golden and Ydstie, 1989) via the use of an ARX model in conjunction with a physical model. This article also included experimental validation.

The implications for chemical processes are clear. In many sectors, such as the petrochemical and biotechnological industries, it is recognized that even small performance improvements in chemical-process control systems can result in substantial economic benefits. The potential benefits of extremum-seeking techniques in the maximization of biomass production rate in well-mixed biological processes has been demonstrated by Krstic (2000) and Krstic and Wang (2000).

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In this study, we investigate an alternative extremum seeking scheme for continuous stirred-tank bioreactors. There are several differences between the proposed design and the scheme studied in Wang et al. (1999). The approach of Krstic (2000), Krstic and Wang (2000), and Wang et al. (1999) does not require the exact knowledge of the objective function and system dynamics. In contrast, the proposed scheme utilizes an explicit structure information of the objective function that depends on system states and unknown plant parameters. Second, the approach developed in Krstic (2000), Krstic and Wang (2000), and Wang et al. (1999) guarantees the global convergence of extremum seeking for the linear system only. For nonlinear systems, only local results are obtained for the convergence and stability of the closed-loop extremum-seeking system (Krstic and Wang, 2000). Since the scheme presented in this article is based on Lyapunov's stability theorem, the global stability is ensured during the seeking of the extremum of the nonlinear continuous stirred-tank bioreactors. It is shown that once a certain level of the persistence of excitation (PE) condition is satisfied, the convergence of the extremum-seeking mechanism can be guaranteed. The article is organized as follows. The following section presents some notation and the problem formulation. In the third section, a parameter-estimation algorithm is developed. The fourth section presents the adaptive extremum-seeking controller and the stability and convergence of the closed-loop extremum-seeking system. Numerical simulations are shown in the fifth section, followed by a brief conclusion in the last section.

Problem Formulation

Consider the following microbial growth models

$$\dot{x} = \mu(x, s)x - ux \tag{1}$$

$$\dot{s} = -k_1 \mu(x, s)x + u(s_0 - s) \tag{2}$$

$$y = k_2 \mu(x, s)x, \tag{3}$$

where states $x \in [0, +\infty)$ and $s \in [0, +\infty)$ denote biomass and substrate concentrations, respectively, $u \geq 0$ is the dilution rate, y is the production rate of the reaction product, s_0 denotes the concentration of the substrate in the feed, and k_1, $k_2 > 0$ are yield coefficients. We consider the case where only s and y are measurable, the biomass concentration x is not available for feedback control [see Zhang et al. (2001) for the case when s and x are measurable].

The nonlinear function $\mu(x, s)$ denotes the growth rate of the process. System comprising Eqs. 1–3 can represent a large class of biochemical processes, depending on the choice of the growth rate $\mu(x, s)$ (Aborhey and Williamson, 1978; Holmberg and Ranta, 1982; Bastin and Dochain, 1990; Boskovic, 1995). There are many different models for $\mu(x, s)$ proposed in the literature (Spriet, 1982; Bastin and Dochain, 1990), for example,

$$\mu(x,s) = \mu(s) = \frac{\mu_m s}{K_s + s} \quad \text{(Monod)} \tag{4}$$

$$\mu(x,s) = \frac{\mu_m s}{K_c x + s} \quad \text{(Contois)} \tag{5}$$

$$\mu(x,s) = \frac{\mu_m K_0 s}{1 + K_1 s + K_2 s^2} \quad \text{(Haldane)}, \tag{6}$$

where $\mu_m > 0$ is the maximum value of the specific growth rate, and positive constant K_s, K_c and K_0 to K_2 denote the coefficients for different growth-rate models.

In this work, we consider the extremum seeking problem for a plant (Eqs. 1–3) with growth rate $\mu(x, s)$ expressed by Monod's model (Eq. 4). The Monod's model is one of the most commonly used models for growth kinetics. However, the scheme developed in this article is not limited to this model and can be easily extended to the plants with other growth-rate representations. The control objective is to design a controller, u, such that the production rate, y, achieves its maximum.

First we calculate the system's equilibria corresponding to a constant dilution rate, u_e. By setting the righthand side of Eqs. 1 and 2 to zero, we obtain two equilibria. The first is $x_e = 0$ and $s_e = s_0$, which is called the washout equilibrium, and is obviously not of practical interest. The second is

$$s_e = \frac{K_s u_e}{\mu_m - u_e}, \quad x_e = \frac{s_0 - s_e}{k_1}.$$

At the steady state, the production rate can be expressed by

$$y_e(s_e) = \frac{k_2 \mu_m s_e (s_0 - s_e)}{k_1 (K_s + s_e)}. \tag{7}$$

From Eqs. 2 and 4, we have

$$\frac{\partial y_e(s_e)}{\partial s_e} = \frac{-k_2 \mu_m}{k_1 (K_s + s_e)^2} \left(s_e^2 + 2K_s s_e - s_0 K_s \right) \tag{8}$$

and

$$\frac{\partial^2 y_e(s_e)}{\partial s_e^2} = \frac{-2k_2 \mu_m}{k_1 (K_s + s_e)^3} \left(K_s^2 + s_0 K_s \right). \tag{9}$$

It is easy to see that $(\partial^2 y_e(s_e))/(\partial s_e^2) < 0$, $\forall s_e \geq 0$. Hence, at the system equilibrium, $y_e(s)$ has a maximum

$$y^* = y_e(s^*) = \frac{k_2 \mu_m s^* x^*}{K_s + s^*}, \tag{10}$$

with

$$s^* = \sqrt{K_s^2 + s_0 K_s} - K_s \tag{11}$$

$$x^* = \frac{s_0 - s^*}{k_1}. \tag{12}$$

From the preceding analysis, we know that if the substrate concentration, s, can be stabilized at the setpoint, s^*, then the production rate, y, is maximized. However, since the exact values of the Monod's model parameters, K_s and μ_m, are usually unknown or at least badly known (Bastin and Dochain, 1990), the desired setpoint, s^*, is not available. In this work, an adaptive extremum-seeking algorithm is developed to search this unknown setpoint such that the production rate, y, is optimized.

Assumption 1. The upper bound of K_s is known, that is, $K_s \leq K_{s0}$, with known constant $K_{s0} > 0$.

As we shall see later, the preceding assumption is important to avoid singularities in the control algorithm.

Parameter Estimation

In this section, we develop the parameter-estimation algorithm for the unknown parameters k_1/k_2, K_s, and μ_m. It follows from Eq. 3 that $\mu(s)x = y/k_2$. Equations 1 and 2 can be reexpressed as

$$\dot{x} = \frac{1}{k_2} y - ux \tag{13}$$

$$\dot{s} = -\frac{k_1}{k_2} y + u(s_0 - s). \tag{14}$$

By Eqs. 3 and 4 and 13 and 14, the time derivative of y is

$$\dot{y} = \frac{k_2 K_s \mu_m x}{(K_s + s)^2} \dot{s} + k_2 \mu(s) \left[\frac{1}{k_2} y - ux \right].$$

Since the biomass concentration x is not measurable, we reexpress $\dot{y}$ by replacing x with $y/k_2 \mu(s)$ as follows

$$\dot{y} = \frac{K_s y}{s(K_s + s)} \left[-\frac{k_1}{k_2} y + u(s_0 - s) \right] + \frac{\mu_m s y}{K_s + s} - uy$$

$$= -uy + \frac{\mu_m s^2 y - \frac{k_1 K_s}{k_2} y^2 + K_s u y (s_0 - s)}{s(K_s + s)} \tag{15}$$

For convenience we shall use the following reformulation of the parameters, $\theta = [\theta_s \theta_\mu \theta_k]^T$ with

$$\theta_\mu = \frac{\mu_m}{K_s}, \quad \theta_s = \frac{1}{K_s}, \quad \theta_k = \frac{k_1}{k_2}. \tag{16}$$

Indeed this allows us to simplify the writing of the equations, since Eqs. 14 and 15 can be rewritten as

$$\dot{s} = -\theta_k y + u(s_0 - s) \tag{17}$$

$$\dot{y} = -uy + \frac{\theta_\mu s^2 y - \theta_k y^2 + (s_0 - s)uy}{s(1 + \theta_s s)}. \tag{18}$$

Let $\hat{\theta}$ denote the estimate of the true parameter θ, and $\hat{s}$ and $\hat{y}$ be the predictions of s and y, respectively, by using the estimated parameter $\hat{\theta}$. The predicted states $\hat{s}$ and $\hat{y}$ are generated by

$$\dot{\hat{s}} = -\hat{\theta}_k y + u(s_0 - s) + k_s e_s \tag{19}$$

$$\dot{\hat{y}} = -uy + \frac{\hat{\theta}_\mu s^2 y - \hat{\theta}_k y^2 + (s_0 - s)uy}{s\left(1 + \hat{\theta}_s s\right)} + k_y e_y, \tag{20}$$

with k_s, $k_y > 0$, and the prediction errors $e_s = s - \hat{s}$ and $e_y = y - \hat{y}$. It follows from Eqs. 17–20 that

$$\dot{e}_s = -k_s e_s - \tilde{\theta} y \tag{21}$$

$$\dot{e}_y = -k_y e_y + \frac{\theta_\mu s^2 y - \theta_k y^2 + (s_0 - s)uy}{s(1 + \theta_s s)}$$

$$- \frac{\hat{\theta}_\mu s^2 y - \hat{\theta}_k y^2 - (s_0 - s)uy}{s\left(1 + \hat{\theta}_s s\right)}$$

$$= -k_y e_y + \frac{\tilde{\theta} \Phi\left(s, y, \hat{\theta}\right) y}{(1 + \theta_s s)\left(1 + \hat{\theta}_s s\right)}, \tag{22}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ and $\Phi(s, y, \hat{\theta}) = [\phi_s \phi_\mu \phi_k]^T$, with

$$\phi_s = -(s_0 - s)u - \hat{\theta}\mu s^2 + \hat{\theta}_k y$$

$$\phi_\mu = \left(1 + \hat{\theta}_s s\right) s$$

$$\phi_k = -\left(1 + \hat{\theta}_s s\right) \frac{y}{s}.$$

By $\theta_s = 1/K_s$, the desired setpoint (Eq. 11) can be reexpressed as $s^* = (\sqrt{1 + s_0 \theta_s} - 1)/\theta_s$. Since the parameter, θ_s, is unknown, we first design a controller to make the substrate concentration, s, follow $(\sqrt{1 + s_0 \hat{\theta}_s} - 1)/\hat{\theta}_s$ that is an estimate of s^*. Later, an excitation signal is designed and injected into the adaptive system such that the estimated parameter $\hat{\theta}_s$ converges to its true value. The extremum-seeking control objective can be achieved when the substrate concentration, s, stabilized at the optimal operating point s^*.

Define

$$z_s = s - \frac{1}{\hat{\theta}_s} \left(\sqrt{1 + s_0 \hat{\theta}_s} - 1 \right) + d(t), \tag{23}$$

where $d(t) \in C^1$ is an excitation signal that will be assigned later. We consider a Lyapunov function candidate

$$V = \frac{z_s^2}{2} + \frac{1}{2} \left(\frac{\tilde{\theta}_\mu^2}{\gamma_\mu} + \frac{\tilde{\theta}_s^2}{\gamma_s} + \frac{\tilde{\theta}_k^2}{\gamma_k} \right) + \frac{e_s^2}{2} + (1 + \theta_s s) \frac{e_y^2}{2}, \tag{24}$$

with constants γ_μ, γ_s, $\gamma_k > 0$. Taking the time derivative of V, we have

$$\dot{V} = z_s \left[\beta\left(\hat{\theta}_s\right) \dot{\hat{\theta}}_s + \dot{d}(t) + \dot{s} \right] + \frac{\tilde{\theta}_\mu \dot{\tilde{\theta}}_\mu}{\gamma_\mu} + \frac{\tilde{\theta}_s \dot{\tilde{\theta}}_s}{\gamma_s} + \frac{\theta_k \dot{\tilde{\theta}}_k}{\gamma_k}$$

$$+ e_s \dot{e}_s + (1 + \theta_s s) e_y \dot{e}_y + \frac{\theta_s}{2} \dot{s} e_y^2, \tag{25}$$

where

$$\beta\left(\hat{\theta}_s\right) = \frac{2 + s_0 \hat{\theta}_s}{2\hat{\theta}_s^2 \sqrt{1 + s_0 \hat{\theta}_s}} - \frac{1}{\hat{\theta}_s^2}. \tag{26}$$

Substituting Eq. 17 and Eqs. 21 and Eqs. 22 into Eq. 25, leads to

$$\begin{aligned}
\dot{V} = & z_s\left[\beta\left(\hat{\theta}_s\right)\dot{\hat{\theta}}_s + \dot{d}(t) - \theta_k y + u(s_0 - s)\right] - \frac{\tilde{\theta}_\mu \dot{\hat{\theta}}_\mu}{\gamma_\mu} \\
& - \frac{\tilde{\theta}_s \dot{\hat{\theta}}_s}{\gamma_s} - \frac{\tilde{\theta}_k \dot{\hat{\theta}}_k}{\gamma_k} - \tilde{\theta}_k y e_s - k_s e_s^2 + \frac{\tilde{\theta}^T \Phi(s, y, \hat{\theta}) y e_y}{1 + \hat{\theta}_s s} \\
& - k_y(1 + \theta_s s)e_y^2 + \frac{\theta_s}{2}\left[-\theta_k y + u(s_0 - s)\right]e_y^2 \\
\leq & z_s\left[\beta\left(\hat{\theta}_s\right)\dot{\hat{\theta}}_s + \dot{d}(t) - \hat{\theta}_k y + u(s_0 - s)\right] \\
& - \tilde{\theta}_k y\left[y\left(\frac{e_y}{s} + z_s\right) + e_s\right] + \theta_\mu y s e_y + \frac{\hat{\theta}_s \phi_s y e_y}{1 + \hat{\theta}_s s} \\
& - \frac{\tilde{\theta}_\mu \dot{\hat{\theta}}_\mu}{\gamma_\mu} - \frac{\tilde{\theta}_s \dot{\hat{\theta}}_s}{\gamma_s} - \frac{\tilde{\theta}_k \dot{\hat{\theta}}_k}{\gamma_k} \\
& - k_s e_s^2 - (1 + \theta_s s)\left[k_y - \frac{\theta_s(s_0 - s)u}{2(1 + \theta_s s)}\right]e_y^2.
\end{aligned} \tag{27}$$

We consider the following parameter updating laws

$$\dot{\hat{\theta}}_s = \begin{cases} \dfrac{\gamma_s \phi_s y e_y}{1 + \hat{\theta}_s s}, & \text{if } \hat{\theta}_s > 1/K_{s0} \\ & \text{or } \hat{\theta}_s = 1/K_{s0} \text{ and } \phi_s y e_y \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{28}$$

$$\dot{\hat{\theta}}_\mu = \gamma_\mu s y e_y$$

$$\dot{\hat{\theta}}_k = -\gamma_k y\left(\frac{y}{s}e_y + z_s + e_s\right), \tag{30}$$

with the initial condition $\hat{\theta}_s(0) \geq 1/K_{s0} > 0$. The role of Assumption 1 clearly appears from Eq. 28 in order to avoid singularity. The tuning law (Eq. 28) is a projection algorithm. In the case where $\hat{\theta}_s > 1/K_{s0}$ or $\hat{\theta}_s = 1/K_{s0}$ and $\phi_s y e_y \geq 0$, the projection algorithm (Eq. 28) suggests that $\dot{\hat{\theta}}_s = (\gamma_s \phi_s y e_y)/(1 + \hat{\theta}_s s)$, which implies

$$\tilde{\theta}_s\left(\frac{\phi_s y e_y}{1 + \hat{\theta}_s s} - \frac{\dot{\hat{\theta}}_s}{\gamma_s}\right) = 0. \tag{31}$$

For the case of $\tilde{\theta}_s = 1/K_{s0}$ and $\phi_s y e_y < 0$, the learning algorithm expressed in Eq. 30 leads to $\dot{\hat{\theta}}_s = 0$, which means that

$$\tilde{\theta}_s\left(\frac{\phi_s y e_y}{1 + \hat{\theta}_s s} - \frac{\dot{\hat{\theta}}_s}{\gamma_s}\right) = \frac{\theta_s \phi_s y e_y}{1 + \hat{\theta}_s s} < 0. \tag{32}$$

Combining Eqs. 31 and 32, we know that

$$\tilde{\theta}_s\left(\frac{\phi_s y e_y}{1 + \hat{\theta}_s s} - \frac{\dot{\hat{\theta}}_s}{\gamma_s}\right) \leq 0. \tag{33}$$

It can also be seen from Eq. 28 that $\hat{\theta}_s(0) \geq 1/K_{s0}$ and $\dot{\hat{\theta}}_s > 0$ for $\hat{\theta}_s = 1/K_{s0}$. Hence, $\hat{\theta}_s \geq 1/K_{s0} > 0$ for all times. Substituting the updating laws (Eqs. 28–30) into Eq. 27, we obtain

$$\begin{aligned}
\dot{V} \leq & z_s\left\{\gamma_s \beta_a\left(y, s, \hat{\theta}_s\right)e_y + \dot{d}(t) - \hat{\theta}_k y \right. \\
& \left. + \left[1 + \gamma_s \beta_b\left(y, s, \hat{\theta}_s\right)e_y\right]u(s_0 - s)\right\} \\
& - k_s e_s^2 - (1 + \theta_s s)\left[k_y - \frac{\theta_s(s_0 - s)u}{2(1 + \theta_s s)}\right]e_y^2,
\end{aligned} \tag{34}$$

where

$$\beta_a\left(y, s, \hat{\theta}_s\right) = \begin{cases} \dfrac{\left(-\hat{\theta}_\mu s^2 + \hat{\theta}_k y\right)y}{1 + \hat{\theta}_s s}\beta\left(\hat{\theta}_s\right), & \text{if } \hat{\theta}_s > 1/K_{s0} \\ & \text{or } \hat{\theta}_s = 1/K_{s0} \\ 0, & \text{and } \phi_s y e_y > 0 \\ & \text{otherwise} \end{cases} \tag{35}$$

$$\beta_b\left(y, s, \hat{\theta}_s\right) = \begin{cases} -\dfrac{y}{1 + \hat{\theta}_s s}\beta\left(\hat{\theta}_s\right), & \text{if } \hat{\theta}_s > 1/K_{s0} \\ & \text{or } \hat{\theta}_s = 1/K_{s0} \\ 0, & \text{and } \phi_s y e_y \geq 0 \\ & \text{otherwise.} \end{cases} \tag{36}$$

Extremum-Seeking Controller

Considering the following extremum-seeking controller

$$u = -\frac{\gamma_s \beta_a\left(y, s, \hat{\theta}_s\right)e_y + \dot{d}(t) - \hat{\theta}_k y + k_z z_s}{\left[1 + \gamma_s \beta_b\left(y, s, \hat{\theta}_s\right)e_y\right](s_0 - s)}, \tag{37}$$

with $k_z > 0$ and the gain function

$$k_y = k_{y0} + \frac{(s_0 - s)K_{s0}|\mu|}{2(1 + K_{s0}s)}, \tag{38}$$

with $k_{y0} > 0$, we have

$$\dot{V} \leq -k_z z_s^2 - k_s e_s^2 - k_{y0} e_y^2. \tag{39}$$

In order to avoid the singularity that may happen to the controller (Eq. 37) when $1 + \gamma_s \beta_b(y, s, \hat{\theta}_s)e_y$ approaches zero, a small learning gain γ_s should be used such that

$$1+\gamma_s \beta_b\left(y, s, \hat{\theta}_s\right) e_y>0. \tag{40}$$

By applying the LaSalle−Yoshizawa's Theorem (Krstic et al., 1995), it is concluded that $\hat{\theta}$, z_s, e_s, and e_y are bounded, and

$$\lim_{t \to \infty} z_s=0, \qquad \lim_{t \to \infty} e_s=0, \qquad \lim_{t \to \infty} e_y=0. \tag{41}$$

It should be noticed that the convergence of the state errors, e_s and e_y, does not mean that the estimated parameters converge to their true values as $t \to \infty$. In the following, we investigate the condition that guarantees the parameter convergence.

By LaSalle's Invariance Principle (Krstic et al., 1995), the error vector $(z_s, e_s, e_y, \tilde{\theta})$ converges to the largest invariant set, M, of the dynamic system (Eqs. 21 and 22) and (Eqs. 28−30) contained in the set $E=\{(z_s, e_s, e_y, \tilde{\theta}) \in R^5 | z_s=e_s=e_y=0\}$. The purpose of the following is to study the invariant set M to obtain the condition under which parameter convergence can be achieved. Since e_s and e_y converge to zero, we know that $\int_0^\infty \dot{e}_s \, dt=e_s(\infty)-e_s(0)=-e_s(0)$ and $\int_0^\infty \dot{e}_y \, dt=e_y(\infty)-e_y(0)=-e_y(0)$. This implies that $\dot{e}_s$ and $\dot{e}_y$ are integrable. It follows from the error equations (Eqs. 21 and 22) that $\ddot{e}_s$ and $\ddot{e}_y$ are functions of y, s, $\hat{y}$, $\hat{s}$, $\hat{\theta}$, d and its time derivatives. Since $\hat{\theta}$, e_s, $e_y \in L_\infty$, and the excitation signals d and $\dot{d}$ are bounded, we know that $\ddot{z}_x$ and $\ddot{e}_y$ are bounded. This implies the uniform continuity of $\dot{e}_s$ and $\dot{e}_y$. By Barbalat's Lemma (Ioannou and Sun, 1996), we conclude that $\dot{e}_s$, $\dot{e}_s \to 0$ as $t \to \infty$.

On the invariant set, M, we have $e_s=e_y \equiv 0$ and $\dot{e}_s=\dot{e}_y \equiv 0$. By setting $e_s=e_y=\dot{e}_s=\dot{e}_y=0$, Eqs. 21 and 22 lead to $\tilde{\theta}_k y=0$ and

$$\frac{\tilde{\theta}^T \Phi\left(s, y, \hat{\theta}\right) y}{\left(1+\theta_s s\right)\left(1+\hat{\theta}_s s\right)}=0, \qquad \left(z_s, e_s, e_y, \tilde{\theta}\right) \in M \tag{42}$$

Since $s>0$ and $\hat{\theta}$ are bounded, we know that

$$\tilde{\theta}_a^T \Phi_a\left(s, y, \hat{\theta}\right) y=0, \qquad \left(z_s, e_s, e_y, \tilde{\theta}\right) \in M, \tag{43}$$

where $\tilde{\theta}_a=[\tilde{\theta}_s \ \tilde{\theta}_\mu]^T$ and $\Phi_a(s, y, \hat{\theta})=[\phi_s \ \phi_\mu]^T$. Therefore, the largest invariant set, M, in E is

$$M=\left\{\left(z_s, e_s, e_y, \tilde{\theta}\right) \in R^6 | z_s=e_s=e_y=0, \right.$$
$$\left. \tilde{\theta}_a^T \Phi_a\left(s, y, \hat{\theta}\right) y=0, \ \tilde{\theta}_k y=0\right\}.$$

It follows from Eq. 43 that

$$\tilde{\theta}_a^T \Phi_a\left(s, y, \hat{\theta}\right) \Phi_a^T\left(s, y, \hat{\theta}\right) y^2 \tilde{\theta}_a=0,$$
$$\forall\left(z_s, e_s, e_y, \tilde{\theta}\right) \in M. \tag{44}$$

If $\Phi_a(s, y, \hat{\theta}) \Phi_a^T(s, y, \hat{\theta}) y^2$ is positive definite, then we can conclude that $\tilde{\theta}=0$. However, it is impossible to satisfy this condition, because the matrix $\Phi_a(s, y, \hat{\theta}) \Phi_a^T(s, y, \hat{\theta}) y^2$ is singular at any given time. We consider the integrals of $\Phi_a(s, y, \hat{\theta}) \Phi_a^T(s, y, \hat{\theta})(s, y, \hat{\theta}) y^2$ for $t \to \infty$. It follows from Eq. 44 that

$$\lim_{t \to \infty} \frac{1}{T_0} \int_t^{t+T_0}\left[\tilde{\theta}_a^T \Phi_a\left(s, y, \hat{\theta}\right) \Phi_a^T\left(s, y, \hat{\theta}\right) y^2 \tilde{\theta}_a\right] d\tau=0, \tag{45}$$

with positive constant T_0. It is shown from Eqs. 28−30 and $\lim_{t \to \infty} e_s$, $e_y=0$, that $\lim_{t \to \infty} \dot{\hat{\theta}}=0$, which implies that $\tilde{\theta}$ converges to a constant when $t \to \infty$. Therefore,

$$\tilde{\theta}_a^T\left\{\lim_{t \to \infty} \frac{1}{T_0} \int_t^{t+T_0} \Phi_a\left(s, y, \hat{\theta}\right) \Phi_a^T\left(s, y, \tilde{\theta}\right) y^2 \, d\tau\right\} \tilde{\theta}_a=0,$$
$$\forall\left(z_s, e_s, e_y, \tilde{\theta}\right) \in M. \tag{46}$$

We are now ready to present a persistence-of-excitation condition for parameter convergence. If the dither signal, $d(t)$, is designed such that the following condition holds

$$\lim_{t \to \infty} \frac{1}{T_0} \int_t^{t+T_0} \Phi_a\left(s, y, \hat{\theta}\right) \Phi_a^T\left(s, y, \hat{\theta}\right) y^2 \, d\tau \geq c_0 I \tag{47}$$

for some $c_0>0$, and

$$s \in \Omega_s=\left\{s | s=\frac{1}{\bar{\theta}_s}\left(\sqrt{1+s_0 \bar{\theta}_s}-1\right)-d(t), \quad \bar{\theta}_s \geq 1/K_{s0}\right\}, \tag{48}$$

then, the parameter error $\tilde{\theta}$ converges to zero asymptotically.

Theorem 1: For the system expressed in Eqs. 1−3, if

(1) The learning rate γ_s is chosen small enough such that Eq. 40 holds, and

(2) The dither signal, $d(t)$, satisfies the PE condition (Eq. 47), then, the extremum-seeking controller (Eq. 37) with adaptive laws (Eqs. 28−30) guarantees that the production rate y converges to an adjustable neighborhood of its maximum y^*.

Proof. Since the PE condition (Eq. 47) is satisfied, we have $\lim_{t \to \infty} \hat{\theta}_s=\theta_s$ and $\lim_{t \to \infty} \hat{\theta}_k=\theta_k$. By $\lim_{t \to \infty} z_s=0$ and $\lim_{t \to \infty} e_y=0$, we see from Eqs. 23 and 37 that

$$\lim_{t \to \infty} s=\frac{1}{\theta_s}\left(\sqrt{1+s_0 \theta_s}-1\right)-\lim_{t \to \infty} d(t)=s^*-\lim_{t \to \infty} d(t) \tag{49}$$

$$\lim_{t \to \infty} u=\lim_{t \to \infty} \frac{\theta_k y-\dot{d}(t)}{s_0-s}. \tag{50}$$

Hence, by Eqs. 3 and 13 we know that when $t \to \infty$, the following equation holds:

$$\dot{x}=\frac{1}{k_2} y-\frac{\theta_k y-\dot{d}(t)}{s_0-s} x=\left[s_0-s+\frac{\dot{d}(t)}{\mu(s)}-k_1 x\right] \frac{\mu(s) x}{s_0-s}.$$

From Eqs. 12 and 49, the preceding equation can be further expressed as follows:

$$\dot{x} = \left[x^* + \frac{d(t)}{k_1} + \frac{\dot{d}(t)}{k_1 \mu(s)} - x \right] \frac{k_1 \mu(s) x}{s_0 - s}.$$

Since x, $\mu(s)$, and $s_0 - s$ are positive definite, we see that (1) $\dot{x} < 0$ when

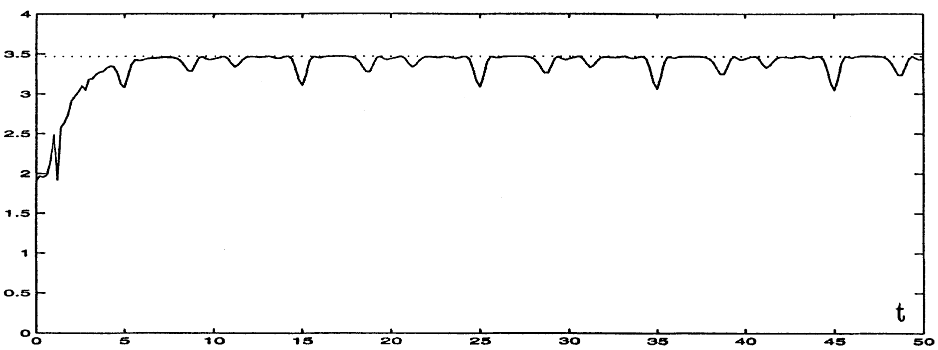
$$x > x^* + \frac{d(t)}{k_1} + \frac{\dot{d}(t)}{k_1 \mu(s)},$$

(2) $\dot{x} > 0$ when

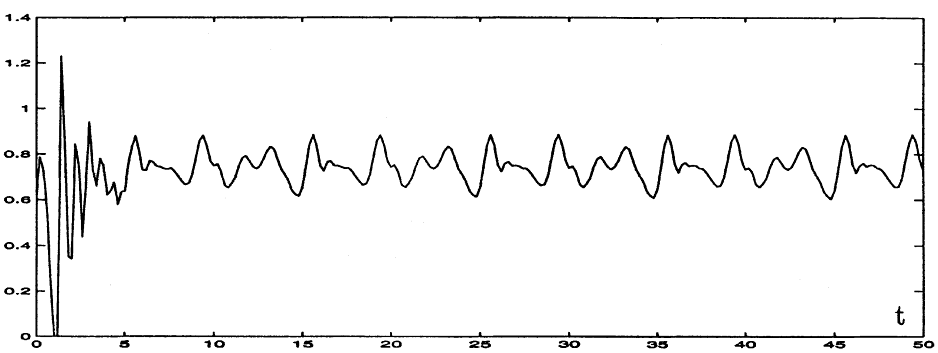
$$x < x^* + \frac{d(t)}{k_1} + \frac{\dot{d}(t)}{k_1 \mu(s)}.$$

This implies that the biomass concentration x converges to the neighborhood of x^*. The size of the neighborhood depends on the external dither signal $d(t)$ and its changing rate. For easy presentation, we denote

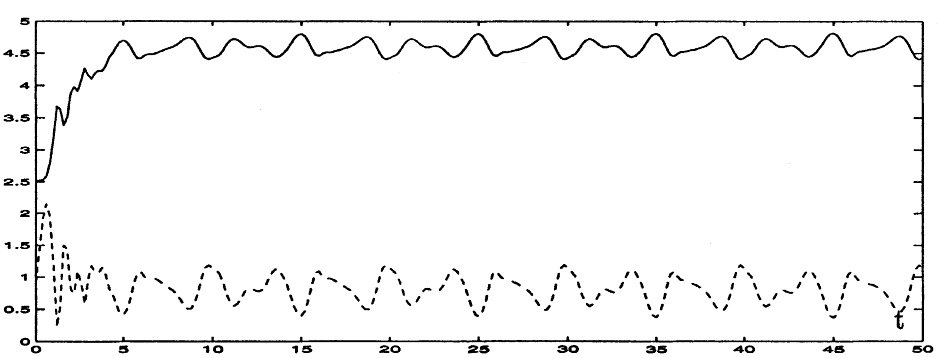
$$\lim_{t \to \infty} x = x^* + \epsilon(d, \dot{d}), \tag{51}$$



(a) Production rate y ("—") and its maximum y^* ("- -")



(b) Control input $u(t)$



(c) States x ("—") and s ("- -")

Figure 1. Adaptive extremum seeking control.

where

$$\epsilon(d, \dot{d}) = \frac{d(t)}{k_1} + \frac{\dot{d}(t)}{k_1 \mu(s)} \tag{52}$$

represents the effect of the dither signal. It is clear that $\epsilon(d, \dot{d}) \to 0$ when $d(t)$, $\dot{d}(t) \to 0$.

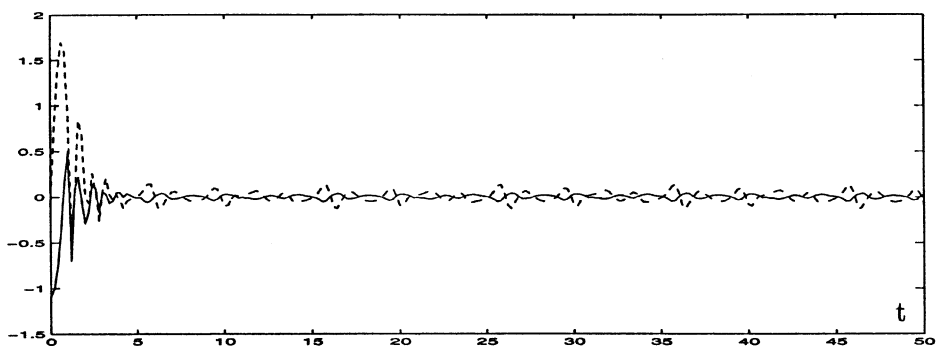
Using the Mean Value Theorem (Ortega and Rheinboldt (1970), we can reexpress the production rate y in Eq. 3 as

$$y = k_2 \mu(s^*) x + k_2 (s - s^*) x \int_0^1 \frac{\partial \mu(s_\lambda)}{\partial s_\lambda} d\lambda,$$

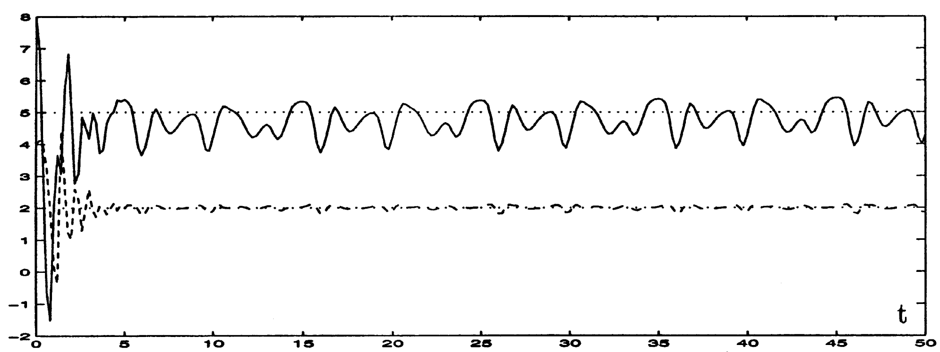
where $s_\lambda = \lambda s + (1 - \lambda) s^*$. Considering Eqs. 10, 49, and 51, we have

$$\lim_{t \to \infty} y = y^* + k_2 \mu(s^*) \epsilon(d, \dot{d}) - \lim_{t \to \infty} \left[k_2 d(t) \int_0^1 \frac{\partial \mu(s_\lambda)}{\partial s_\lambda} d\lambda \right]. \tag{53}$$

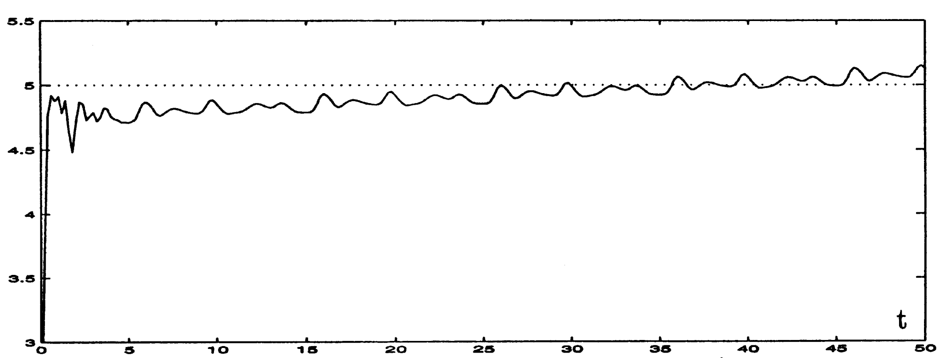
The preceding equation implies that the production rate, y, converges to a neighborhood of the desired production rate



(d) State prediction errors e_y ("—") and e_s ("- -")



(e) Parameter θ_μ ("...") and its estimate $\hat{\theta}_\mu$ ("—")
Parameter θ_k ("...") and its estimate $\hat{\theta}_k$ ("- -")



(f) Parameter θ_s ("...") and its estimate $\hat{\theta}_s$ ("- -")

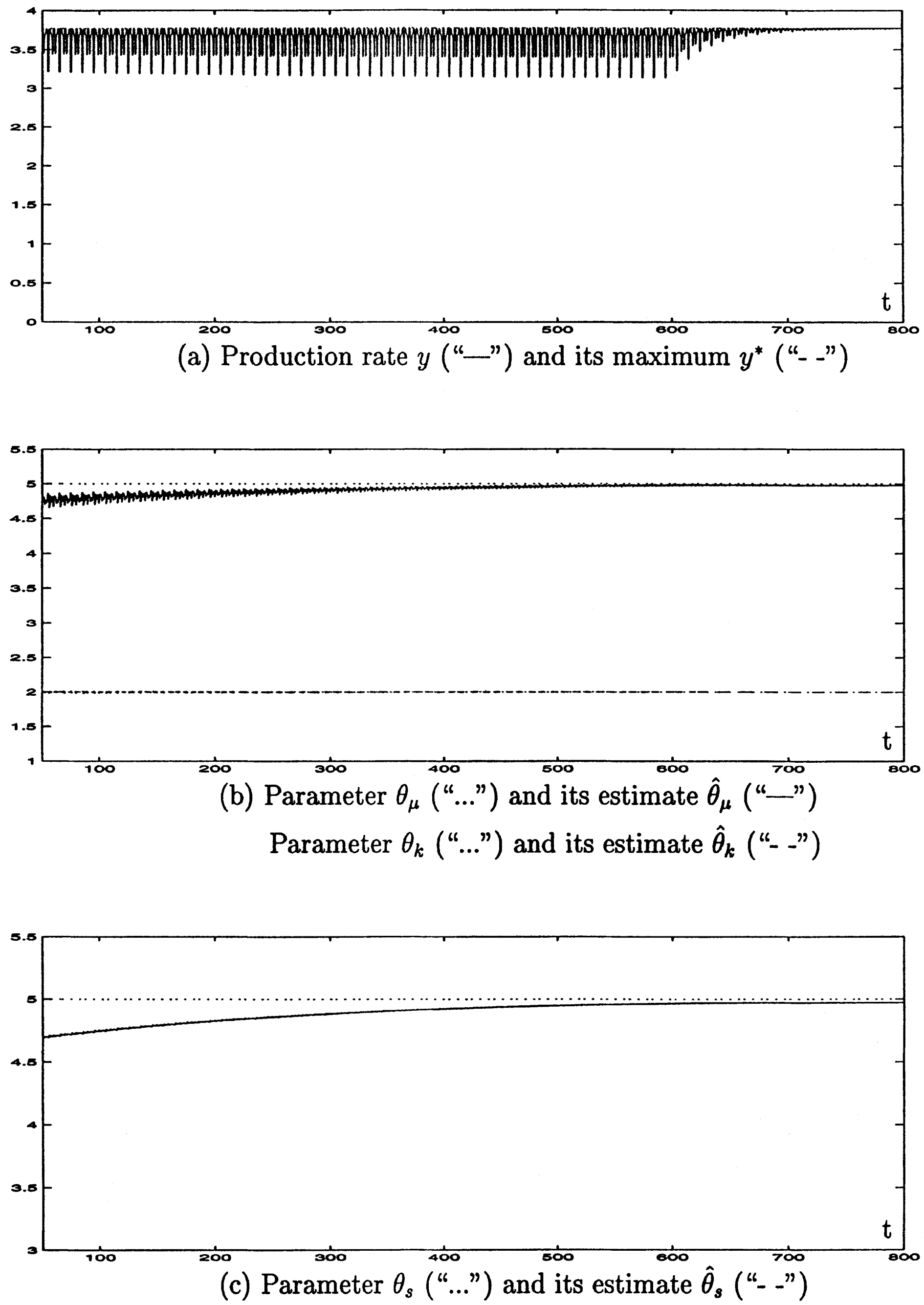
Figure 1. Adaptive extremum seeking control, (continued).

y^*, whose size is adjustable by tuning the amplitudes of the injected dither signal $d(t)$ and its time derivative. Q.E.D.

Remark 1. Theorem 1 shows that the excitation signal $d(t)$ plays an important role in solving the extremum-seeking problem. In general, the higher the amplitude of the dither signal, the faster the optimal operation point can be found. However, Eq. 53 indicates that it also introduces a disturbance to the system. To reduce the effect of this disturbance, it is necessary to use the small amplitude, $d(t)$, in practical applications. Although this article considers the biochemical processes represented by the Monod's growth model, the proposed extremum-seeking scheme is ready to be extended to the plants with other growth-rate models, such as Contois' model (Eq. 5) or the Haldane's model (Eq. 6) (Zhang et al., 2002) by using the similar design procedure proposed in this article.

Simulation Results

To show the effectiveness of the proposed design, a simulation study is performed using the experimental conditions provided in Wang et al. (1999). The following parameters and



(a) Production rate y ("—") and its maximum y^* ("- -")

(b) Parameter θ_μ ("...") and its estimate $\hat{\theta}_\mu$ ("—")
Parameter θ_k ("...") and its estimate $\hat{\theta}_k$ ("- -")

(c) Parameter θ_s ("...") and its estimate $\hat{\theta}_s$ ("- -")

Figure 2. Parameter convergence of the adaptive extremum-seeking controller.

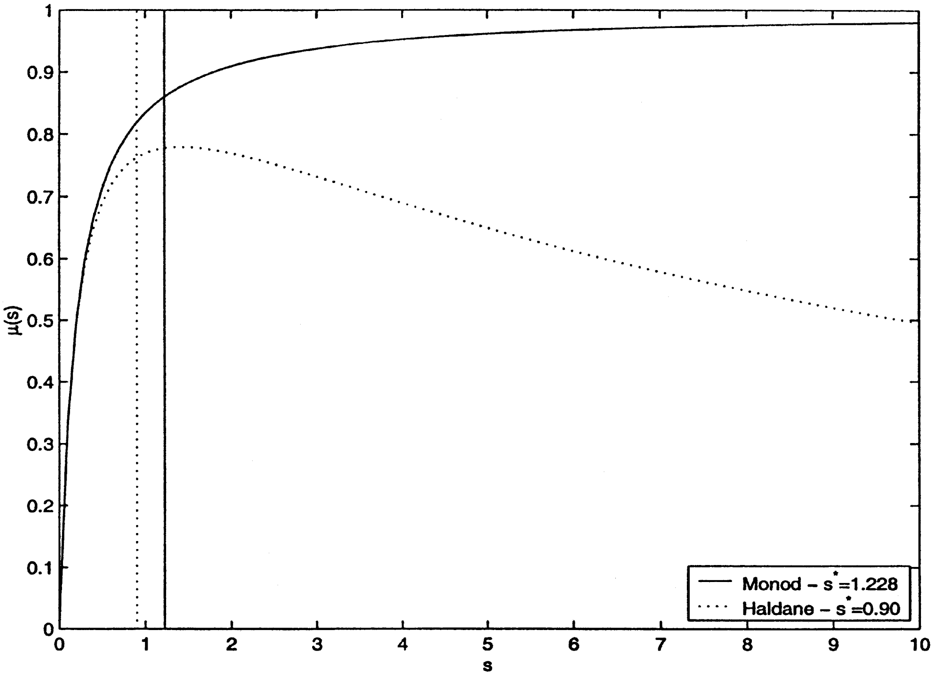


Figure 3. Growth rate and position of maximum growth rate for the Monod kinetics (——) and the Haldane kinetics (. . .).

initial states are used in the simulation experiment:

$$K_s = 0.2, \quad \mu_m = 1.0, \quad Y = 0.5, \quad k_1 = 2.0, \quad k_2 = 1.0,$$
$$s_0 = 10.0, \qquad x(0) = 3.0, \qquad s(0) = 0.9.$$

We suppose that the upper bound of K_s is known as $K_{s0} = 0.5$. The design parameters in the adaptive controller (Eq. 37) and the adaptive laws (Eqs. 28–30) are

$$\gamma_s = 2.0, \qquad \gamma_\mu = 20.0, \qquad \gamma_k = 2.0$$
$$\hat{\theta}_s(0) = 8.0, \quad \hat{\theta}_\mu(0) = 2.0, \quad \hat{\theta}_k(0) = 4.0.$$

The dither signal is chosen as $d(t) = 2.2 - \cos(0.5t) - \cos(0.3t)$.

Figure 1 presents the simulation result of the adaptive extremum-seeking controller. Figure 1a shows that the production rate reaches its maximum value 3.77 after $t = 6$. Due to the injection of the excitation signal, $d(t)$, the production rate keeps oscillating around the optimal point. Figures 1b and 1c plot the time evolutions of the control input and the system states. We see that even though the prediction errors in Figure 1d are close to zero at time $t = 50$, the estimated parameters $\hat{\theta}_\mu$ and $\dot{\theta}_s$ do not converge to the true values. The convergence of the algorithm can be confirmed by inspection of a simulation performed over a longer time period (that is, $t = 800$). In order to remove the effect of the excitation signal $d(t)$, we let $d(t)$ vanish exponentially as $t > 600$. The production rate is shown in Figure 2a. The estimated parameters and their true values are shown in Figure 1e and 1f for $t = 50$ and in Figures 2b and 2c for $t = 800$. The simulation confirms that the production-rate converges to its maximum, and that the convergence of the estimated parameters is achieved. However, parameter convergence remains prohibitively slow in this case. This phenomenon highlights the inherent difficulty associated with the estimation of K_s and μ_m, as discussed in Zhang and Guay (2001).

In most applications, the exact growth-rate kinetics are not known. Since the proposed optimization scheme relies explicitly on the assumption of Monod kinetics, it is imperative that its robustness to changes in the kinetics be tested. To this end, the proposed scheme was applied to a bioreactor process model with Haldane kinetics. The kinetics are given by

$$\mu(s) = \frac{\mu_m s}{K_s + s + K_i s^2}, \tag{54}$$

where $\mu_m = 1$, $K_s = 0.2$ and $K_i = 0.1$. In order to demonstrate the impact of this unknown nonlinearity, the growth rate for the Monod kinetics (with $\mu_m = 1$ and $K_s = 0.2$) and the Haldane kinetics are shown in Figure 3. The significant difference in growth rate is observed. The position of the corresponding optimum is also shown. The Haldane kinetics provides an optimum production rate of 3.46 compared to the value of 3.77 expected from the Monod kinetics.

The result of the application of the proposed extremum-seeking controller is shown in Figure 4. Figure 4a gives the resulting production rate. The corresponding control action and the process states are given in Figures 4b and 4c. Despite the significant difference in process kinetics, the closed-loop system behaves in a manner that is almost identical to the nominal case. The maximum production rate of 3.46 is recovered by the control scheme. The simulation results seem to

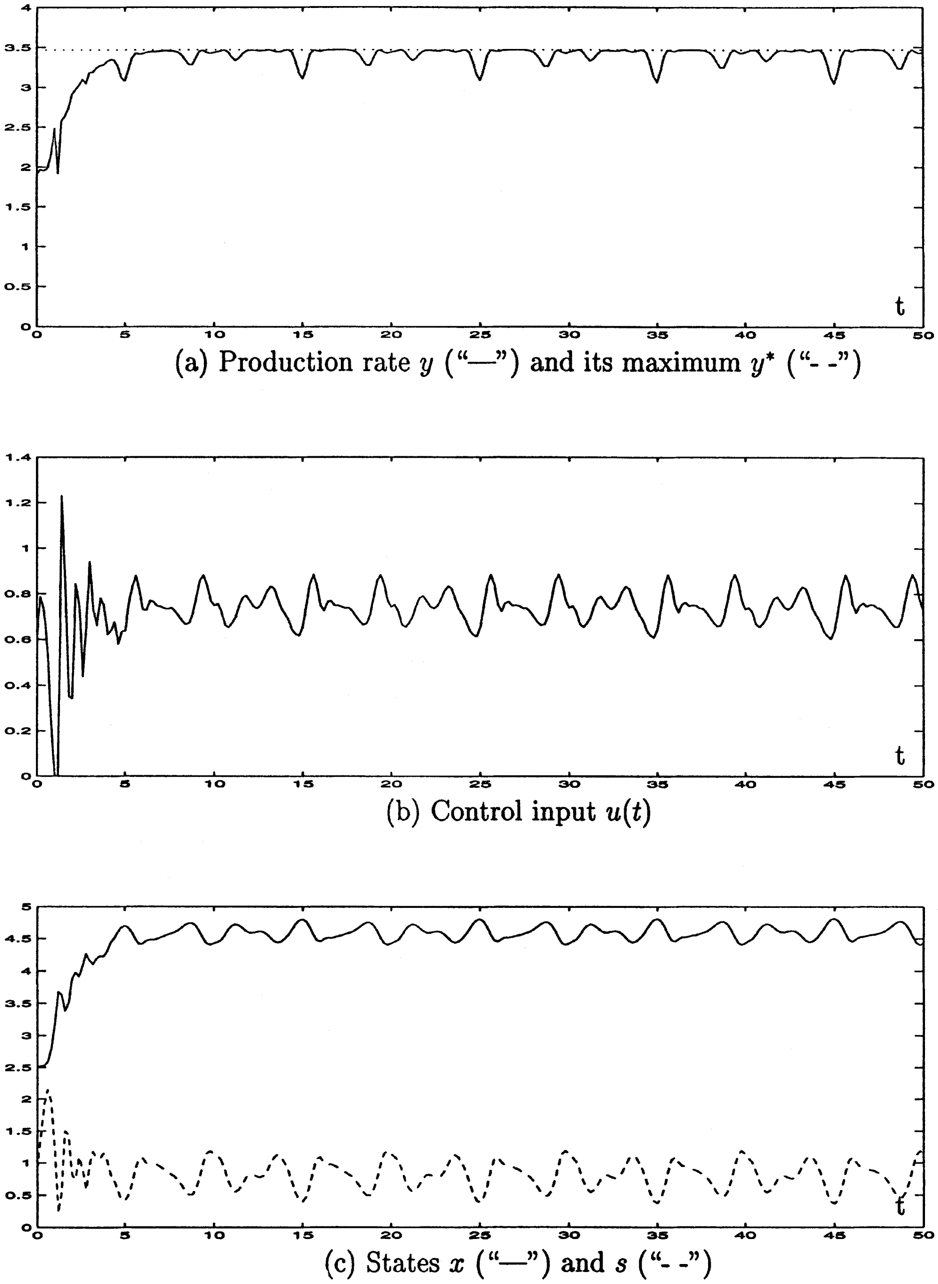


Figure 4. Robustness of the proposed adaptive extremum-seeking control to kinetic model mismatch.

indicate that the technique can be robust to modeling uncertainty. However, more research is required to investigate the robustness of this scheme in a general framework.

Conclusion

We have solved a class of extremum-seeking control problem for continuous stirred-tank bioreactors represented by Monod's growth model with unknown parameters. The proposed extremum-seeking controller drives biomass and substrate concentrations to unknown desired setpoints that optimize the production rate. It has been shown that when the external dither signal is designed such that the persistent excitation condition is satisfied, the proposed adaptive extremum-seeking controller guarantees the convergence of the production rate of the bioreactor to the neighborhood of its maximum.

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